## Making Sense of High throughput Protein-Protein Interaction Data

A Graph Theoretic Algorithm for AP-MS Data

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Auckland

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## Which <br> proteins are these?

Graphic courtesy of:
U.S. Department of Energy Human Genome Program http://www.ornl.gov/hgmis


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## Two Types of Data: Pairwise Protein Relationships

- AP-MS (Affinity Purification - Mass Spectrometry )
- Measures Complex Comembership
- Gavin, et al. (Nature, 2002)
- TAP : Tandem Affinity Purification
- Ho, et al. (Nature, 2002)
- HMS-PCI: High-throughput Mass Spectromic Protein Complex Identification
- Y2H (Yeast Two Hybrid)
- Measures Physical Interactions
- Ito, et al. (PNAS, 1998)
- Uetz, et al. (Nature, 2000)


## AP-MS




Using a bait protein, AP-MS technology finds hit proteins that are comembers of at least one complex with the bait.

Y2H technology finds pairs of physically interacting proteins.


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AP-MS data:


Y2H data:

*Estimation of A requires estimation of $K$, the number of complexes.


We want to estimate the bipartite protein complex membership graph, A:

AP-MS data: $\quad$ Y2H data:

*Estimation of A requires estimation of $K$, the number of complexes.

## Existing analyses of AP-MS data

- Gavin, et al.
- Functional organization of the yeast proteome by systematic analysis of protein complexes (Nature 2002)
- Purifications grouped together based on significant overlap (p.143)
- Bader and Hogue
- Analyzing Yeast Protein-Protein Interaction Data Obtained from Different Sources (Nature Biotechnology, 2002)
- An Automated Method for Finding Molecular Complexes in Large Protein Interaction Networks (Bioinformatics 2003)
- Works within the realm of pairwise interactions without recognition of the bipartite graph structure for complex membership
- "Spoke" and "Matrix" models
- Treat AP-MS data as "hypothetical pairwise interactions"
- Jansen, et al.
- A Bayesian Networks Approach for Predicting Protein-Protein Interactions from Genomic Data (Science 2003)
- Deals with pairwise complex comemberships, not comprehensive complex membership


## Four Unique Aspects to our Algorithm

1. Some proteins participate in more than one complex
2. In an AP-MS experiment, some proteins are used as baits and some proteins are only ever found as hits
3. Graph theoretic paradigm to allow for succinct expression of constructs involved

- Bipartite graph for complex membership (A)
- Relationship of complex membership (A) to complex comembership $(Y)$ assayed in an AP-MS experiment ( $Z$ )
- $\quad$ AP-MS and Y 2 H are different technologies that measure different relationships between proteins

4. Statistical paradigm to allow for false positive and false negative observations
5. Some proteins participate in more than one complex

## PP2A

## Heterotrimeric complex consisting of:

## Tpd3

- regulatory A subunit


## Rts1 or Cdc55

- regulatory B subunits


## Pph21 or Pph22

- catalytic subunits

Jiang and Broach (1999). EMBO.

Gavin, et al. (2002)
Rgraphviz plot of yTAP C151

## Bader \& Hogue (2002)

Portion of Figure 2:
Overlap of the spoke models
of TAP and HMS-PCI.

Jansen, et al. (2003) PIT Bayesian Network, LR>600 central node=Tpd3 YCR002C=Cdc10, YJR076C=Cdc11, YMR109W=Myo5

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Zds1 and Zds2 (known cell-cycle regulators) only exist in complexes with the Cdc55-Pph22 trimer!

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2. In an AP-MS experiment, some proteins are used as baits and some proteins are only ever found as hits

## Supplementary Material S1. List of all purifications.

Note that frequently found proteins are omitted from this list (see Table S2)

| $\#$ | Tagged <br> protein | Proteins found |
| :--- | :--- | :--- |
| 1 | Abd1 | Abd1 Rpb2 Spt5 |
| 2 | Acc1 | Acc1 Cct5 Sit4 YLR386W |
| 3 | Ade1 | Ade1 |
| 4 | Ade12 | Ade12 |
| 5 | Ade13 | Ade13 Prt1 |
| 6 | Ade4 | Ade4 Cys3 Rna1 |
| 7 | Ade5,7 | Ade5,7 |
| 8 | Ade6 | Ade6 |
| 9 | Adk1 | Adk1 |
| 10 | Ado1 | Ado1 |
| 11 | Akl1 | Akl1 |
| 12 | Aos1 | Adh1 Aos1 Uba2 Yef3 |
| 13 | Apc2 | Apc1 Apc2 Cdc16 Cdc23 Cdc27 |
| 14 | Apd1 | Apd1 |
| 15 | Apg14 | Vma1 Vps30 |
| 16 | Apl2 | Apl2 Apl4 Apm1 Apm2 Aps1 Mis1 Rpa135 |
| 18 | Apl5 | Apl5 Apl6 Apm3 Aps3 Ckb1 |
| 18 | Apl6 | Apl5 Apl6 Apm3 Eno2 |
| 19 | Apm3 |  |
| 20 | Apm3 | Apl6 Apm3 |
| 2 | Apra | A.pl2 |

## Subgraph of $\boldsymbol{Z}$



Raw TAP purifications (Gavin et al.)
Available at http://www.nature.com
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| 2 | Apr2 |  |

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| 20 | Apm3 |  |
| 2 | Arc2 | Apt2 |
|  |  |  |

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3. Graph theoretic paradigm to allow for succinct expression of constructs involved

- Bipartite graph for complex membership
-Relationship of complex membership (A) to complex comembership (Y) assayed in an AP-MS experiment (Z)
-AP-MS and Y2H are different technologies that measure different relationships between proteins
We want to estimate A using AP-MS assays of $Y$.
i) True Complex $\quad$ ii) PCMG Physical Topology
ail
(asi

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The Connection: Maximal Complete Subgraphs
Complete Subgraph: set of $n$ nodes for which all $n(n-1)$ directed edges exist Maximal Complete Subgraph: complete subgraph that is not contained in any other complete subgraph
3. Graph theoretic paradigm to allow for succinct expression of constructs involved
-Relationship of complex membership (A) to complex comembership (Y) assayed in an AP-MS experiment ( $Z$ )

Y represents "ideal" complex comembership observations from perfectly sensitive and perfectly specific AP-MS technology. Y depends on the baits that are used in an experiment. $Y$ is assayed by AP-MS technology.

| a-1) | a-II) | a-III) | a-IV) |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| b-1) | b-II) | b-III) | b-IV) |
|  |  |  |  |
| c-1) | c-11) | C-III) | c-IV) |
|  |  | $\text { Pcc } \rightarrow \text { (P) }$ <br> (ब) (a) |  |
| d-l) | d-II) | d-III) | d-IV) |
|  |  |  |  |

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|  |  |  |  |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |

The Connection: Maximal BH-Complete Subgraphs
BH-Complete Subgraph: set of $n$ bait nodes and $m$ hit-only nodes for which all $n(n-1)+n m$ directed edges exist $\dagger$
Maximal BH-Complete Subgraph: BH-complete subgraph that is not contained in any other complete subgraph
4. Statistical paradigm to allow for false positive and false negative observations

Z represents actual observations using AP-MS technology.

| a) $Y_{\left\{P_{1}, P_{2}, P_{3}\right\}}$ | b) $Z_{\left\{P_{1}, P_{2}, P_{3\}} \text { : false negative }\right.}$ edges between P1-P3, P2-P4 | c) $Z_{\left\{P_{1}, P_{2}, P_{3}\right\}}$ : false positive edge between P3 and P7 |
| :---: | :---: | :---: |
| d) true $A$ | e) false negative edge in $A$ between P6 and C1 <br> observed $Z_{\left\{P_{1},\right.} P_{2}, P_{3\}}$ | f) false positive edge in A between P7 and C1 <br> observed $Z_{\left\{P_{1}, P_{2}, P_{3}\right\}}$ |

4. Statistical paradigm to allow for false positive and false negative observations

Z represents actual observations using AP-MS technology.

We will look for sets of proteins that form maximal BH -complete subgraphs with an allowance for false positive and false negative observations.
a) $Y_{\left\{P_{1}, P_{2}, P_{3}\right.}$
d) true $A$

## Our Goal

- for any (every) organism or tissue type we want to estimate the complex membership graph
- that is, the bipartite graph where one set of nodes are all proteins and the other are all complexes
- we are limited by the experimental data, experimental techniques and models


## Graphs as Matrices


\(\mathrm{A}=\begin{gathered} <br>
\mathrm{P}_{\mathrm{c} 1} <br>
\mathrm{P}_{\mathrm{c} 2} <br>
\mathrm{P}_{\mathrm{c} 3} <br>
\mathrm{P}_{\mathrm{c} 4} <br>
\mathrm{P}_{\mathrm{c} 5} <br>

\mathrm{P}_{\mathrm{c} 6}\end{gathered}\)| $\mathrm{C}_{7}$ | $\mathrm{C}_{8}$ |
| :---: | :---: |
| 1 | 1 |
| 1 | 0 |
| 0 | 1 |
| 1 | 0 |
| 0 |  |

$$
\mathrm{Z}=
$$



## Graphs as Matrices


\(\mathrm{A}=\begin{gathered} <br>
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| :---: | :---: |
| 1 | 1 |
| 1 | 0 |
| 0 | 1 |
| 1 | 0 |

$$
\mathrm{Z}=
$$



## In summary...

$A \xrightarrow[Y=A \otimes A^{\prime}]{ } Y$
AP-MS data for $N$ bait proteins and $M$ hit-only proteins


We start with an initial estimate for $A$, and then refine that estimate according to a two component probability measure:

$$
P(Z \mid A, \mu, \alpha)=L\left(Z \mid Y=A \otimes A^{\prime}, \mu, \alpha\right) C(Z \mid A, \mu, \alpha)
$$

## $P(Z \mid A, \mu, \alpha)=L\left(Z \mid Y=A \otimes A^{\prime}, \mu, \alpha\right) C(Z \mid A, \mu, \alpha)$

$L$ is the usual likelihood for independent Bernoulli observations of the existence of an edge under a logistic regression model with user-specified values of $\mu$ and $\alpha$.

$$
\begin{gathered}
L\left(Z \mid A \otimes A^{\prime}, \mu, \alpha\right)=\prod_{i=1}^{N} \prod_{j=1, j \neq i}^{N} p_{i j} Z_{i j}\left(1-p_{i j}\right)^{\left(1-Z_{i j}\right)} \prod_{l=1}^{N} \prod_{m=N+1}^{N+M} p_{l m} Z_{l m}\left(1-p_{l m}\right)^{\left(1-Z_{l m}\right)} \\
\quad p_{i j}=\operatorname{Pr}\left(Z_{i j}=1 \mid \mu, \alpha, Y_{i j}\right), \text { and } \log \left(\frac{p_{i j}}{1-p_{i j}}\right)=\mu+\alpha Y_{i j} \\
\text { sensingly tested edges }
\end{gathered}
$$

## $P(Z \mid A, \mu, \alpha)=L\left(Z \mid Y=A \otimes A^{\prime}, \mu, \alpha\right) C(Z \mid A, \mu, \alpha)$

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\quad p_{i j}=\operatorname{Pr}\left(Z_{i j}=1 \mid \mu, \alpha, Y_{i j}\right), \text { and } \log \left(\frac{p_{i j}}{1-p_{i j}}\right)=\mu+\alpha Y_{i j} \\
\text { sensitivity tested edgested edges }
\end{gathered}=\frac{\mathrm{e}^{\mu}}{1+\mathrm{e}^{\mu}}, \quad \text { specificity }=\frac{\mathrm{e}^{\mu+\alpha}}{1+\mathrm{e}^{\mu+\alpha}} .
$$

Using $L$, we can estimate $Y_{i j}=0$ or 1 for $i=1, \ldots, N$ and $j=1, \ldots, N+M$. For $i=j, Y_{i j}=Y_{j i}$.

## $P(Z \mid A, \mu, \alpha)=L\left(Z \mid Y=A \otimes A^{\prime}, \mu, \alpha\right) C(Z \mid A, \mu, \alpha)$

Assumptions for $\mu$ and $\alpha$ in our analyses:

1) $\operatorname{Pr}\left(Z_{i j}=0 \mid \mu, \alpha, Y_{i j}=0\right)>.5$ and $\operatorname{Pr}\left(Z_{i j}=1 \mid \mu, \alpha, Y_{i j}=1\right)>.5$
-sensitivity and specificity are greater than .5
2) $\operatorname{Pr}\left(Z_{i j}=0 \mid \mu, \alpha, Y_{i j}=1\right)>\operatorname{Pr}\left(Z_{i j}=1 \mid \mu, \alpha, Y_{i j}=0\right)$
-false negative probability is greater than false positive probability

Under these assumptions for $\mu$ and $\alpha, L$ is easily maximized.
For singly tested bait-hit pairs, $\quad \hat{Y}_{i j}=Z_{i j}$.
For doubly tested bait-bait pairs, $\left(\hat{Y}_{i j}, \hat{Y}_{j i}\right)=\max \left(Z_{i j}, Z_{j i}\right)$.

## $P(Z \mid A, \mu, \alpha)=L\left(Z \mid Y=A \otimes A^{\prime}, \mu, \alpha\right) C(Z \mid A, \mu, \alpha)$

Assumptions for $\mu$ and $\alpha$ in our analyses:

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[^0]
## Given Y , What is A ? Identifiability

$$
\begin{aligned}
& \qquad Y=A \otimes A^{\prime} \\
& Y \text { is uniquely determined by } A \text {, } \\
& \text { but } A \text { is not uniquely determined by } Y .
\end{aligned}
$$

$$
\begin{aligned}
& \text { One Trimer } \\
&
\end{aligned}
$$

One Trimer with a
Dimer Subcomplex

$$
\left.A=\begin{array}{l|l|} 
& C_{1} \\
P_{1} & C_{2} \\
P_{1} & 1 \\
\hline
\end{array} \right\rvert\, \begin{array}{ll}
1 \\
P_{2} & 1 \\
P_{3} & 1
\end{array} 0
$$

| Three Dimers |  |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ |
| $\mathrm{P}_{1}$ | 1 | 1 | 0 |
| $A=P_{2}$ | 1 | 0 | 1 |
| $\mathrm{P}_{3}$ | 0 | 1 | 1 |

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& Y \text { is uniquely determined by } A, \\
& \text { but } A \text { is not uniquely determined by } Y \text {. }
\end{aligned}
$$

One Trimer

$A=$|  | $C_{1}$ |
| :--- | :--- |
| $\mathrm{P}_{1}$ | 1 |
| $\mathrm{P}_{2}$ | 1 |
| $\mathrm{P}_{3}$ | 1 |

One Trimer with a Dimer Subcomplex

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P_{1} & C_{2} \\
P_{2} & 1 \\
\hline
\end{array} \right\rvert\, \begin{array}{ll}
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P_{3} & 1 \\
P_{3} & 1
\end{array} 0
$$

$A=$| $\mathrm{P}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ |  |
| :--- | :---: | :---: | :---: |
| $\mathrm{P}_{2}$ | 1 | 1 | 0 |
| $\mathrm{P}_{2}$ | 1 | 0 | 1 |
| $\mathrm{P}_{3}$ | 0 | 1 | 1 |
|  |  |  |  |

$A$ is identifiable if it assumed to consist of maximal subgraphs of $Y$. I.e., given the $Y$ above, we would find the "one trimer" version of $A$.

## Initial Estimate of $A$




## Initial Estimate of $A$



|  |  | $\mathrm{P}_{\mathrm{c} 1}$ | $\mathrm{P}_{\mathrm{c} 2}$ | $\mathrm{P}_{\mathrm{c} 3}$ | $\mathrm{P}_{\mathrm{c}}$ | $\mathrm{P}_{\text {c } 5}$ | $\mathrm{P}_{\text {c6 }}$ |  |  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 1 | 0 | 1 | 0 | 1 |  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 0 |
|  | $\mathrm{P}_{\mathrm{c} 3}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $A=P_{c 3}$ | 0 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{ct}}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 5}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\text {c }}$ | 0 | 1 |
|  | $\mathrm{P}_{\text {c } 6}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\text {c }}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 1}$ | $\mathrm{P}_{\mathrm{c} 2}$ | $\mathrm{P}_{\mathrm{c} 3}$ | $\mathrm{P}_{\text {c }}$ | $\mathrm{P}_{\mathrm{c} 5}$ | $\mathrm{P}_{66}$ |  |  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ |
| $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 |
| $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 1 | 0 | 1 | 0 | 1 |  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 0 | 0 |
| $\mathrm{P}_{\mathrm{c} 3}$ | 1 | 0 | 1 | 0 | 1 | 0 | \# | $A=P_{c 3}$ | 0 | 1 | 0 |
| $\mathrm{P}_{\mathrm{ct}}$ | 1 | 1 | 0 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 0 | 1 |
| $\mathrm{P}_{\text {c5 }}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\text {c } 5}$ | 0 | 1 | 1 |
| $\mathrm{P}_{66}$ | 1 | 1 | 0 | 1 | 1 | 1 |  | $\mathrm{P}_{66}$ | 1 | 0 | 1 |

## Initial Estimate of $A$



|  |  | $\begin{aligned} & P_{c 4} \\ & P_{c 5} \\ & P_{c 6} \end{aligned}$ | 1 1 1 | 1 1 1 | $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ | 1 1 1 | 1 1 1 | $\begin{aligned} & 1 \\ & 1 \\ & 1 \end{aligned}$ |  | $\mathrm{P}_{\text {c4 }}$ $\mathrm{P}_{65}$ $\mathrm{P}_{66}$ | 1 1 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - |  | $\mathrm{P}_{\mathrm{c} 1}$ | $\mathrm{P}_{\mathrm{c} 2}$ | $\mathrm{P}_{\mathrm{c} 3}$ | $\mathrm{P}_{\text {c }}$ | $\mathrm{P}_{\mathrm{c} 5}$ | $\mathrm{P}_{\mathrm{c} 6}$ |  |  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ |
|  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 1 | 0 | 1 | 0 | 1 |  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 0 |
| $Y=$ | $=P_{\text {c }}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $A=P_{c 3}$ | 0 | 1 |
|  | $\mathrm{P}_{\mathrm{c}}$ |  | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 1 |
|  | $\mathrm{P}_{\text {c } 5}$ |  | 0 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\text {c } 5}$ | 0 | 1 |
|  | $\mathrm{P}_{\text {c6 }}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{66}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 1}$ | $\mathrm{P}_{\mathrm{c} 2}$ | $\mathrm{P}_{\mathrm{c} 3}$ | $\mathrm{P}_{\mathrm{c} 4}$ | $\mathrm{P}_{\text {c }}$ | $\mathrm{P}_{\mathrm{c} 6}$ |  |  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ |
| $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 |
| $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 1 | 0 | 1 | 0 | 1 |  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 0 | 0 |
| $Y=P_{c 3}$ | 1 | 0 | 1 | 0 | 1 | 0 | $\Rightarrow$ | $A=P_{c 3}$ | 0 | 1 | 0 |
| $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 1 | 0 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{ct}}$ | 1 | 0 | 1 |
| $\mathrm{P}_{\text {c }}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\text {c5 }}$ | 0 | 1 | 1 |
| $\mathrm{P}_{\text {c6 }}$ | 1 | 1 | 0 | 1 | 1 | 1 |  | $\mathrm{P}_{66}$ | 1 | 0 | 1 |

## Initial Estimate of $A$



|  |  | $\begin{aligned} & \mathrm{P}_{\mathrm{c}} \\ & \mathrm{P}_{\mathrm{c} 5} \\ & \mathrm{P}_{\mathrm{c}} \end{aligned}$ | 1 | 1 1 1 | 1 1 1 | 1 1 1 | 1 1 1 | $1$ |  | $\mathrm{P}_{\text {c4 }}$ $\mathrm{P}_{\text {c5 }}$ $\mathrm{P}_{\text {c6 }}$ | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{P}_{\mathrm{c} 1}$ | $\mathrm{P}_{\mathrm{c} 2}$ | $\mathrm{P}_{\mathrm{c} 3}$ | $\mathrm{P}_{\mathrm{c} 4}$ | $\mathrm{P}_{\text {c5 }}$ | $\mathrm{P}_{\text {c6 }}$ |  |  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ |
| - | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 1 | 0 | 1 | 0 | 1 |  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 0 |
| Y | $\mathrm{P}_{\mathrm{c} 3}$ | 1 | 0 | 1 | 1 | 1 | 1 | $\Rightarrow$ | $A=P_{c 3}$ | 0 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 5}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 5}$ | 0 | 1 |
|  | $P_{c 6}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\text {c6 }}$ | 1 | 1 |
|  | $\mathrm{P}_{\mathrm{c} 1}$ | $\mathrm{P}_{\mathrm{c} 2}$ | $\mathrm{P}_{\mathrm{c} 3}$ | $\mathrm{P}_{\mathrm{c} 4}$ | $\mathrm{P}_{\text {c5 }}$ | $\mathrm{P}_{\text {c6 }}$ |  |  | $\mathrm{C}_{1}$ | $\mathrm{C}_{2}$ | $\mathrm{C}_{3}$ |
| $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{c} 1}$ | 1 | 1 | 1 |
| $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 1 | 0 | 1 | 0 | 1 |  | $\mathrm{P}_{\mathrm{c} 2}$ | 1 | 0 | 0 |
| $Y=P_{c 3}$ | 1 | 0 | 1 | 0 | 1 | 0 | $\Rightarrow$ | $A=P_{c 3}$ | 0 | 1 | 0 |
| $\mathrm{P}_{\mathrm{c} 4}$ | 1 | 1 | 0 | 1 | 1 | 1 |  | $\mathrm{P}_{\mathrm{ct}}$ | 1 | 0 | 1 |
| $\mathrm{P}_{\mathrm{c} 5}$ | 1 | 0 | 1 | 1 | 1 | 1 |  | $\mathrm{P}_{65}$ | 0 | 1 | 1 |
| $\mathrm{P}_{\text {c } 6}$ | 1 | 1 | 0 | 1 | 1 | 1 |  | $\mathrm{P}_{66}$ | 1 | 0 | 1 |

## Initial Estimate of $A$



## Initial Estimate of $A$



Since we only use a subset of the proteins as baits, we cannot identify maximal complete subgraphs in $Y$. Instead, the initial estimate of A based on $Y$ consists of the maximal BH-complete subgraphs in $Y$.

## Why C? <br> Why isn't L enough?

- At most, each edge is tested twice, and independent errors are made in the observation of all edges.
- A false negative observation from a bait to a hit would break one complex into two estimated complexes.
- Effectively, C relaxes the maximal BH -complete subgraph requirement for the initial complex estimates to accommodate a proportion of false negative observations in accordance with the sensitivity
 of the AP-MS technology.


## $P(Z \mid A, \mu, \alpha)=L\left(Z \mid Y=A \otimes A^{\prime}, \mu, \alpha\right) C(Z \mid A, \mu, \alpha)$

$C$ is designed to allow combinations of the complexes in the estimated $A$ that increase $C$ in favor of small decreases in $L$.
$C(Z \mid A, \mu, \alpha)=\prod_{k=1}^{K} \Phi\left(c_{k}\right) \Gamma\left(c_{k}\right) \quad$ ( $K=$ total \# of complexes)
$c_{k}$ is a complex estimate with $n_{k}$ bait proteins and $m_{k}$ hit - only proteins
$\Phi\left(c_{k}\right)=$ cumulative probability of observing a particular missing edge pattern or something more extreme for the edges in complex $c_{k}$,
i.e. two - sided $p$ - value from Fisher's exact test on node indegree
$\Gamma\left(c_{k}\right)=\binom{t_{k}}{x_{k}} \frac{e^{x_{k}(\mu+\alpha)}}{\left(1+e^{(\mu+\alpha)}\right)^{t_{k}}}, \quad\left(\frac{e^{(\mu+\alpha)}}{1+e^{(\mu+\alpha)}}=\right.$ sensitivity $)$
$t_{k}=n_{k}\left(n_{k}+m_{k}-1\right)=$ number of tested edges in BH - complete subgraph for $c_{k}$ $x_{k}=$ number of observed edges in BH - complete subgraph for $c_{k}$

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$t_{k}=n_{k}\left(n_{k}+m_{k}-1\right)=$ number of tested edges in BH - complete subgraph for $c_{k}$
$x_{k}=$ number of observed edges in BH - complete subgraph for $c_{k}$
Since the thousands of individual edges in $Y$ are tested at most twice, an estimate of A based solely on L may not be accurate. C offers a second criteria to further refine $A$.

## Combining Complex Estimates

For two complex estimates, $c_{k 1}$ and $c_{k 2}$, we check to see if they increase $P$ when treated as one complex $c_{k^{*}}$.

Specifically, if $\log P_{k^{*}}-\log P_{k 1, k 2}>0$, we combine $c_{k 1}$ and $c_{k 2}$ a new $c_{k^{*}}$.

$$
\begin{aligned}
\log P_{k^{*}} \log P_{k 1, k 2} & =\log \Phi\left(c_{k^{*}}\right)-\log \Phi\left(c_{k 1}\right)-\log \Phi\left(c_{k 2}\right) \\
& +\log \Gamma\left(c_{k^{*}}\right)-\log \Gamma\left(c_{k 1}\right)-\log \Gamma\left(c_{k 2}\right) \\
& +\sum_{S_{\text {new }}}\left[\alpha z_{g h}-\log \left(1+e^{\mu+\alpha}\right)+\log \left(1+e^{\mu}\right)\right]
\end{aligned}
$$

where $S_{\text {new }}=$ set of all edges between proteins $g$ and $h$ that are being changed from "absent" to "present"

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$$
\begin{aligned}
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& +\log \Gamma\left(c_{k^{*}}\right)-\log \Gamma\left(c_{k 1}\right)-\log \Gamma\left(c_{k 2}\right) \\
& +\sum_{S_{\text {new }}}\left[\alpha z_{g h}-\log \left(1+e^{\mu+\alpha}\right)+\log \left(1+e^{\mu}\right)\right]
\end{aligned}
$$

where $S_{\text {new }}=$ set of all edges between proteins $g$ and $h$ that are being changed from "absent" to "present"

In general, $P$ increases for a smaller number of complexes that are both reflective of approximate maximal BH -complete subgraph structure and consistent with the observed data.

## Complex Estimation Algorithm

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1. Find the MLE for $Y$ using $Z$.

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1. Find the MLE for $Y$ using $Z$.
2. Find the initial estimate for $A$ by constructing maximal $B H-$ complete subgraphs in $Y$.

## Complex Estimation Algorithm

1. Find the MLE for $Y$ using $Z$.
2. Find the initial estimate for $A$ by constructing maximal $B H-$ complete subgraphs in $Y$.
3. Order the columns of $A$ according to the number of baits.

## Complex Estimation Algorithm

1. Find the MLE for $Y$ using $Z$.
2. Find the initial estimate for $A$ by constructing maximal BH complete subgraphs in $Y$.
3. Order the columns of $A$ according to the number of baits.
4. Set $k=1$ and $K=$ number of columns of $A$.

## Complex Estimation Algorithm

1. Find the MLE for $Y$ using $Z$.
2. Find the initial estimate for $A$ by constructing maximal $\mathrm{BH}-$ complete subgraphs in $Y$.
3. Order the columns of $A$ according to the number of baits.
4. Set $k=1$ and $K=$ number of columns of $A$.
5. For $c_{k}$, find the set $A_{k}$ of columns of $A$, excluding $c_{k}$, that share at least one common entry of " 1 ". Calculate $\log P_{k^{*}}-\log P_{k 1, k 2}$ for $c_{k}$ paired with all elements in $A_{k}$.

## Complex Estimation Algorithm

1. Find the MLE for $Y$ using $Z$.
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6. If at least one value of $\log P_{k^{*}}-\log P_{k 1, k 2}$ is greater than 0 , replace $c_{k}$ with the union of $c_{k}$ and $c_{A k m a x}$ the element of $A_{k}$ giving the largest value of $\log P_{k^{*}}-\log \mathrm{P}_{\mathrm{k} 1, \mathrm{k} 2}$. Remove $c_{\text {Akmax }}$ and any columns that are strictly less than $c_{k} U c_{A k m a x}$. Set $K=$ number of columns of $A$.

## Complex Estimation Algorithm

1. Find the MLE for $Y$ using $Z$.
2. Find the initial estimate for $A$ by constructing maximal $\mathrm{BH}-$ complete subgraphs in $Y$.
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7. If none of the values of $\log P_{k^{\star}}-\log P_{k 1, k 2}$ are greater than 0 , set $k=k+1$, and return to step 5 .

## Complex Estimation Algorithm

1. Find the MLE for $Y$ using $Z$.
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7. If none of the values of $\log P_{k^{*}}-\log P_{k 1, k 2}$ are greater than 0 , set $k=k+1$, and return to step 5 .
8. Repeat until $k=K$.

## Two types of complex estimates to interpret with care



## TAP data analysis

- Sensitivity=.75, Specificity=. 001
- Gene Ontology (GO) cellular component-based similarity measure in an extended logistic regression model
- Purpose is to increase the probability that two proximally located proteins are complex comembers even if there is not an edge between them
- 720 complexes total
- 123 UnRBB
- 331 SBMH


## - 266 multi-bait complexes with at least 2 proteins and at least 2 edges

- Compared these 266 complexes to the 232 yTAP complexes (Gavin et al. 2002) through both a large scale comparison, and complex-by-complex for several complexes.


## Large Scale Comparison to Known Complexes

- Similarity measure: $\omega=\min (\mathrm{i} / \mathrm{a}, \mathrm{i} / \mathrm{b})$
- $\mathrm{a}=$ \# proteins in complex $\mathrm{A}, \mathrm{b}=$ \# proteins in complex B
- $\mathrm{i}=$ \# proteins in both A and B
- Munich Information Center for Protein Sequences (MIPS) reports a list of 267 curated protein complexes , 129 of which involved 595 proteins contained in the TAP data.
- Using $\omega>.70$ as a mapping criteria and the common subset of 595 proteins, we mapped 85 of our complexes to 65 MIPS complexes and 40 yTAP complexes to 32 MIPS complexes.



## Figure 1 from Gavin, et al.




\% YEAST protein complex database
NEW SEARCH HELP \& FAQ CONTACT LOGOUT

Hello Denise Scholtens. You're logged in as dscholte $\square$

## browse complexes.



| P YEAST protein complex database - Microsoft Internet Explorer |  |  | - $\square$ ㅁ $\times$ |
| :---: | :---: | :---: | :---: |
| File Edit View Favorites Tools Help |  |  |  |
|  |  |  |  |
| \| Address http://yeast.cellzome.com/complexd.php?k=3f70f27c0aald |  | - $¢$ Go | Links " |
| :\% YEAST protein complex database |  |  |  |
| Hello Denise Scholtens. You're logged in as dscholte | Search: | submit | ? |

## complex details.

## Complex ID <br> 16 <br> Function <br> Cell cycle

Proteins:

## Description

Acetyl-CoA carbox...Cytoplasmic
GTPase-activating
Component of Cha...Cytoplasmic, Cytoskeletal
Guanine-nucleotid... Plasma membrane
Phosphatidylinosito lysosome/vacuole
Negative regulator,
Mitochondrial ribos...Mitochondrial
Sit4p-associated p...
Protein that associ..
Protein that associ.
Protein serine/thre...Cytoplasmic
Protein of unknow...
Putative membran...Unspecified membrane Protein of unknow...
Serine/threonine p..

* this flag assigns to proteins which have been used as baits in our purifications



## Gavin, yTAP C121




## Arp2/3

## Arp2/3 complex:

Arp2
Arp3
Arc15
Arc18
Arc19
Arc35
Arc40
'The Arp2/3 complex is a stable multiprotein assembly required for the nucleation of actin filaments in all eukaryotic cells and consists of seven proteins in human and yeast.'

Winter, et al (1997). Curr Biol. Higgs and Pollard (2001). Annu Rev Biochem.


## Arp2/3

Arp2/3 complex:
Arp2
Arp3
Arc15
Arc18
Arc19
Arc35
Arc40


## Origin Recognition Complex

| Origin |
| :--- |
| Recognition |
| Complex: |
|  |
| Orc1 |
| Orc2 |
| Orc3 |
| Orc4 |
| Orc5 |
| Orc6 |
| Dutta and Bel (1997). Annu |
| Rev Cell Dev Biol. |



## Origin Recognition Complex



## Exosome

Exosome:
Rrp4
Rrp41 (Ski6)
Rrp42
Rrp43
Rrp44 (Dis3)
Rrp45
Rrp46
Mtr3
Rrp40
Csl4
Rrp6 (only in nuclear
exosome)

Allmang, et al (1999). Genes Devel.


## Exosome

| Exosome: |
| :--- |
|  |
| Rrp4 |
| Rrp41 (Ski6) |
| Rrp42 |
| $\operatorname{Rrp43}$ |
| $\operatorname{Rrp44}$ (Dis3) |
| $\operatorname{Rrp45}$ |
| $\operatorname{Rrp46}$ |
| Mtr3 |
| $\operatorname{Rrp40}$ |
| $\operatorname{CsI4}$ |
| $\operatorname{Rrp6}$ (only in nuclear |
| exosome) |



## Exosome

Exosome:
Rrp4
Rrp41 (Ski6)
Rrp42
Rrp43
Rrp44 (Dis3)
Rrp45
$\operatorname{Rrp46}$
Mtr3
$\operatorname{Rrp40}$
Cs14
$\operatorname{Rrp6}$ (only in nuclear
exosome)

Allmang, et al (1999). Genes Devel.


## PP2A

## Tpd3

Heterotrimeric complex consisting of:

- regulatory A subunit

Cdc55 or Rts1

- regulatory B subunits

Pph21 or Pph22

- catalytic subunits

Jiang and Broach (1999). EMBO.


## PP2A

## Heterotrimeric

 complex consisting of:
## Tpd3

- regulatory A subunit


## Rts1 or Cdc55

- regulatory B subunits

Pph21 or Pph22

- catalytic subunits

Jiang and Broach (1999). EMBO.


## RNA Polymerases I, II and III



## RNA Polymerases I, II and III



RNA Polymerases I and III


## RNA Polymerase II



## mRNA cleavage and polyadenylation



## mRNA cleavage and polyadenylation

| CF I: | PF I: |
| :--- | :--- |
|  |  |
| Rna14 | Cft1 |
| Rna15 | Cft2 |
| Pcf11 | Ysh1 (Brr5) |
| ClpI | Pta1 |
| Hrp1 | Fip1 |
|  | Pfs2 |
|  | Yth1 |
|  | YKL059C (Mpe1) |
|  | YGR156W (Pti1) |
|  | Pap1 |

-Hrp1 is CFIB - a separate component that shuttles between the nucleus and cytoplasm -CF II is Cft1, Cft2, Ysh1, Pta1 -Yeast requires the cooperativity of CFI \& PFI
-Pfs2 and Rna14 exhibit an in vitro interaction

Gross and Moore (2001). PNAS.
Zhao, et al (1997). J Biol Chem.
Skaar and Greenleaf (2002) Mol Cell.
Vo, et al (2001). Mol Cell Biol.


## mRNA cleavage and polyadenylation



## TRAPP

## TRAPP: <br> Bet3 <br> Trs20 <br> Bet5 <br> Trs23 <br> Trs33 <br> Trs31 <br> Trs65 (Kre11) <br> Trs85 (Gsg1) <br> Trs120 <br> Trs130

Sacher, et al (2000). EJCB.


## TRAPP

TRAPP:
Bet3
Trs20
Bet5
Trs23
Trs33
Trs31
Trs65 (Kre11)
Trs85 (Gsg1)
Trs120
Trs130

Sacher, et al (2000). EJCB.


## New complexes to Test?



Only complex in our analysis involving these four, except for some SBMH complexes. Currently unreported in the literature.

## New complexes to Test?



YCR072C and Kre32 have no annotation in GO or PubMed.

## New complexes to Test?



These are both undocumented in the literature - note that Enp1, YDL060W (Tsr1), and YNL207W (Rio2) are in both complexes.

## Conclusions

- Distinction between the structures of the graphs representing both the estimation goal and the available data afforded a simple complex membership estimation algorithm allowing multiple complex membership by individual proteins.
- These complex membership estimates allow a more detailed view of complexes than other analyses.


## What's Next?

- New Experiments
- Test previously unidentified complexes
- Mutate a gene and see what happens to its complex composition?
- Coordination with Other Data
- Y2H data to determine physical connectivity of the proteins in a complex
- Cell-cycle gene expression data to determine which complexes function in a cell cycledependent manner, and to determine the expression profile of multi-complex proteins
- Sequence data to determine binding sites


## Thanks to

- Marc Vidal, DFCI
- Very helpful discussions about the biology
- Jeff Gentry, DFCI
- Graph plotting software: Rgraphviz
- Jianhua Zhang, DFCI
- Annotation package: yeast
- Vince Carey, Channing Lab
- Helpful discussion and insights
- Members of Gentleman/Carey Lab


[^0]:    We have an estimate for $Y$, but our goal is to estimate $A$.
    We use the transformation $Y=A \otimes A^{\prime}$ and maximal $B H$-complete subgraphs.

